FUZZY TRANSPORTATION PROBLEM BY USING TRAPEZOIDAL FUZZY NUMBERS

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ABSTRACT *A fuzzy set can be represented mathematically by giving a value representing its grade of membership in the fuzzy set to each possible individual in the universe of discourse. The part of uncertainty cannot be avoided by any branch of science, engineering, medical and management. Transportation problem deals with transportation or distribution of goods or services from several supply origins to several demand destinations and it is the main branch of operation research. In real-world transportation planning, decision problems, input data and related parameters, such as available supply and forecast demand, are often imprecise/fuzzy because some information is incomplete or unavailable. In this article, we have used the fuzzy trapezoidal numbers to obtain a best approximate solution to the fuzzy transportation problem.*

Keywords:*Fuzzy, Trapezoidal, FNWCM, FLCM, FVAM.*

Introduction

In 1947, T.C. Koopmans developed a model called 'optimum utilization of the transportation system'. Due to these two significant contributions, there was a huge improvement in transportation problem, which involved number of trading sources and a number of trading destinations. Every trading source has certain capacity and every destination has a certain demand associated with a certain price of trading from the sources to the destination. The main aim is to minimize the transportation cost while meeting the demands of the destinations. In a fuzzy optimization problem, fuzzy random numbers gives an approximate solution. We are going to use fuzzy trapezoidal random numbers to obtain a best approximate solution to the fuzzy transportation problem. To solve a Mathematical problem which does not have an analytical solution can be solved by using random numbers which is computer-based experiment through the random numbers. the fuzzy programming approach to multi-objective transportation problem was started by Prof. A.K. Bit in year 1993 [5]. Later Chanas S. [6] [7] obtainedan optimal solution to the transportation problem with fuzzy cost coefficients along with this he explained the method of solving the fuzzy integer transportation problem.Dymowa L [8] solved the transportation problem under probabilistic and fuzzy uncertainties.A. NagoorGani [9] applied simplex type algorithm for solving fuzzy transportation problem. S. Nareshkumar[10] used modified Vogel's approximation method to study fuzzy transportation problems.

In this article first defined the α–cut and operations on fuzzy trapezoidal numbers, in the next section, Crisp transportation problem is converted into fuzzy transportation problem by using the trapezoidal fuzzy number. Crisp number x is converted into a trapezoidal fuzzy number using [x-2d, x-d, x+d, x+2d] and solved by using FNWCM, FLCM and FVAM method. Finally, we discuss the result.

Basic Definitions

Fuzzy set:

A fuzzy set is denoted by $\bar{A}$ defined by $\bar{A} = \{(x, \mu_A(x)): x \in A, \mu_A(x) \in [0, 1]\}$. In the pair $(x, \mu_A(x))$, the first element x belong to the crisp set A, the second element $\mu_A(x)$, belong to the interval [0, 1], called Membership function.

The support of a fuzzy set $\bar{A}$ is subset of $\bar{A}$ defined as $\{x \in A: \mu_A(x) > 0\}$

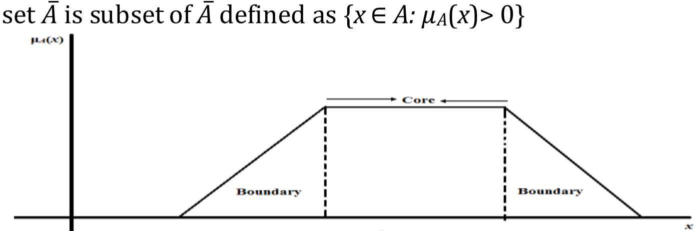

Figure 1: Core, Support, and Boundaries of a fuzzy set

Normality:
If core of fuzzy set is nonempty then that fuzzy set is called normal,other words, if there exist at least one point $x\in X$with $\mu_A(x) = 1$.

***α*–cut:**
α-cut of a fuzzy set $\bar{A}$ is denoted by $\bar{A}_\alpha$and is defined as $\bar{A}_\alpha = \{x\in X : \mu_A(x) \geq \alpha \}$

Defuzzification:
If $\bar{x}$ = (*a*, *b*, *c*, *d*) beany given trapezoidal fuzzy number then we use mean to Defuzzifyi.e. $x= \frac{a+b+c+d}{3}$.
If (-*b*,-*a*, *a*, *b*) is given fuzzy trapezoidal number then its crisp value is zero.

Tabular representation:

Table 1: Fuzzy transportation problem

Destination / Origin	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$	-------	$\bar{D}_n$	Supply
$\bar{O}_1$	$\bar{C}_{11}$	$\bar{C}_{12}$	$\bar{C}_{13}$	-------	$\bar{C}_{1n}$	$\bar{A}_1$
$\bar{O}_2$	$\bar{C}_{21}$	$\bar{C}_{22}$	$\bar{C}_{23}$	-------	$\bar{C}_{2n}$	$\bar{A}_2$
⋮	⋮	⋮	⋮		⋮	⋮
$\bar{O}_m$	$\bar{C}_{m1}$	$\bar{C}_{m2}$	$\bar{C}_{m3}$	-------	$\bar{C}_{mn}$	$\bar{A}_m$
Demand	$\bar{B}_1$	$\bar{B}_2$	$\bar{B}_3$	-------	$\bar{B}_n$	$\sum_{i=1}^{n} \bar{B}_i = \sum_{j=1}^{m} \bar{A}_j$

Trapezoidal fuzzy number:
Another shape of fuzzy number is trapezoidal fuzzy number. This shape is originated from the fact that there are several points whose membership degree is maximum (*α* = 1).
Membership function of trapezoidal fuzzy number $\bar{A}$= (a_1, a_2, a_3, a_4) is interpreted as follows

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \\ 0 & otherwise \end{cases}$$

When$a_2 = a_3$, the trapezoidal fuzzy number coincides with triangular one.

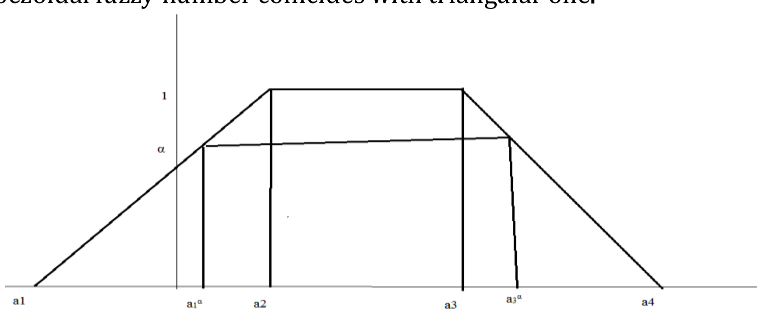

Figure 2: Trapezoidal fuzzy number (a_1, a_2, a_3, a_4)

α- Cut for trapezoidalfuzzy number:
For any α ∈ [0, 1]

$$\frac{a_1^\alpha - a_1}{a_2 - a_1} = \alpha, \frac{a_4 - a_4^\alpha}{a_4 - a_3} = \alpha$$

$a_1^\alpha = (a_2 - a_1)\alpha + a_1 a_4^\alpha = = -(a_4 - a_3)\,\alpha + a_4$
Thus $\bar{A}_\alpha = [a_1^\alpha, a_4^\alpha] = [(a_2 - a_1)\alpha + a_1, -(a_4 - a_3)\,\alpha + a_4]$

Operations ontrapezoidal fuzzy numbers:
Addition, Subtraction and Multiplication of any two trapezoidalfuzzy numbers are alsotrapezoidal

fuzzy number.Suppose trapezoidal fuzzy numbers $\bar{A}$ and $\bar{B}$ are defined as,
$\bar{A} = (a_1, a_2, a_3, a_4)$ and $\bar{B} = (b_1, b_2, b_3, b_4)$

i) Addition: $\bar{A}\ (+)\ \bar{B} = (a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4)$
$= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

ii) Subtraction: $\bar{A}\ (-)\ \bar{B} = (a_1, a_2, a_3, a_4) - (b_1, b_2, b_3, b_4)$
$= (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

iii) Symmetric image: $(-\bar{A}) = (-a_4, -a_3, -a_2, -a_1)$

iv) Multiplication: $\bar{A} \times \bar{B} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4)$

Ordering any twotrapezoidal fuzzy numbers:
To order any two trapezoidal fuzzy numbers $\bar{X} = [a_1, a_2, a_3, a_4]$and $\bar{Y} = [b_1, b_2, b_3, b_4]$we find here α cut say $\bar{X}_\alpha$ and $\bar{Y}_\alpha$. Thus $\bar{X}_\alpha = [a_1^\alpha, a_4^\alpha]$ and $\bar{X}_\alpha = [b_1^\alpha, b_4^\alpha]$
Then $\bar{X} \le \bar{Y}$ if $\int_0^1 (a_1^\alpha + a_4^\alpha + a_1 + a_2)\,d\alpha \le \int_0^1 (b_1^\alpha + b_4^\alpha + b_1 + b_2)\,d\alpha$ otherwise $\bar{Y} \le \bar{X}$

Mat-lab code to order the two fuzzy trapezoidal numbers:

```
clc
'Enter your first trapezoidal fuzzy number\n'
a1=input('');
a2=input('');
a3=input('');
a4=input('');
a=[a1a2a3a4]
 'Enter your second trapezoidal fuzzy number\n'
b1=input('');
b2=input('');
b3=input('');
b4=input('');
b=[b1b2b3b4]
symsx
f1=(a1-a3)+a3*x;
f2=(a2+a4)-a4*x;
f3=f1+f2;
m1=int((f3+a1+a2)*x,0,1);
n1=double(m1)
f4=(b1-b3)+b3*x;
f5=(b2+b4)-b4*x;
f6=f4+f5;
m2=int((f6+b1+b2)*x,0,1);
n2=double(m2)
if(n1<n2)
'a<b'
else
'a>b'
end
```

Numerical example:
Consider the following crisp transportation problem

Table 2: Crisp transportation problem

Origin \ Destination	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$	$\bar{D}_4$	Supply
$\bar{O}_1$	5	2	4	3	22
$\bar{O}_2$	4	8	1	6	15

$\bar{O}_3$	4	6	7	5	8
Demand	7	12	17	9	

Table 3: Comparison of minimum transportation cost

	NWCR	LCM	VAM
Minimum Transportation cost	131	105	104

Table 4: Fuzzy NWCR by using trapezoidal fuzzy number

	Destination				Supply
Origin	[3,4,6,7] **(5,6,8,9)**	[0,1,3,4] **(10,11,13,14)**	[2,3,5,6] **(-4,-1,7,10)**	[1,2,4,5]	(19,20,24,25)
	[2,3,5,6]	[6,7,9,10]	[-1,0,2,3] **(5,9,19,23)**	[4,5,7,8] **(-11,-6,8,13)**	(12,13,17,18)
	[2,3,5,6]	[4,5,7,8]	[5,6,8,9]	[3,4,6,7] **(-6,0,16,22)**	(6,7,9,10)
Demand	(5,6,8,9)	(10,11,13,14)	(15,16,18,19)	(7,8,10,11)	

Minimum Fuzzy Transportation Cost:
[15, 24, 48, 63]+ [0, 11, 39, 56] + [-8, -3, 35, 60] + [-5, 0, 38, 69] + [-44,-30, 56,104] + [-18, 0, 96,154] = **[-60, 2, 312, 506]**
Final computational value of trapezoidalfuzzy number is= **[-16, -9, 9, 16]**= 0

Table 5: Fuzzy LCM by using trapezoidal fuzzy number

	Destination				Supply
Origin	[3,4,6,7]	[0,1,3,4] **(10,11,13,14)**	[2,3,5,6] **(-6,-3,5,8)**	[1,2,4,5] **(7, 8,10,11)**	(19, 20, 24, 25)
	[2,3,5,6]	[6,7,9,10]	[-1,0,2,3] **(12,13,17,18)**	[4,5,7,8]	(12,13,17,18)
	[2,3,5,6] **(5,6,8,9)**	[4,5,7,8]	[5,6,8,9] **(-3,-1,3,5)**	[3,4,6,7]	(6,7,9,10)
Demand	(5,6,8,9)	(10,11,13,14)	(15,16,18,19)	(7,8,10,11)	

Minimum Fuzzy Transportation Cost:
[10, 18, 40,54] + [0, 11, 39, 56] + [-12, -9, 25, 48]+ [7, 16, 40, 55] + [-15, -6, 24, 45] + [-12, 0, 34, 54] = **[-22, 30, 202, 312]**
Final computational value of trapezoidalfuzzy number is= **[-16, -9,9, 16]** *= 0*

Table 6: Fuzzy VAM by using trapezoidal fuzzy number

	Destination				Supply
Origin	[3,4,6,7]	[0,1,3,4] **(10,11,13,14)**	[2,3,5,6] **(-6,-3,5,8)**	[1,2,4,5] **(7,8,10,11)**	(19,20,24,25)
	[2,3,5,6]	[6,7,9,10]	[-1,0,2,3] **(12,13,17,18)**	[4,5,7,8]	(12,13,17,18)
	[2,3,5,6] **(5,6,8,9)**	[4,5,7,8]	[5,6,8,9] **(-3,-1,3,5)**	[3,4,6,7]	(6,7,9,10)
Demand	(5,6,8,9)	(10,11,13,14)	(15,16,18,19)	(7,8,10,11)	

Minimum Fuzzy Transportation Cost: [0, 11, 39,56] + [-12, -9, 25, 48] + [7, 16, 40, 55] + [-12, 0, 34, 54] +[-15, -6, 24, 45] + [10,18, 40, 54] = **[-22, 30, 202, 312]**
Final computational value of trapezoidalfuzzy number is= **[-16, -9, 9, 16]** = 0

Table 7: Fuzzy transportation cost by using trapezoidal fuzzy number

Method	**Fuzzy Solution**	**Defuzzificationby using Mean Method**
Fuzzy North West Corner Method	[-60,2,312,506]	188.75
Fuzzy Matrix Minima Method	[-22,30,202,312]	130.5
Fuzzy VAM	[-22,30,202,312]	130.5

Conclusion:

In this article we have proposed a new method to order any two fuzzy trapezoidal numbers.By using Mat-lab program to order any two fuzzy trapezoidal numbers we have solved the fuzzy transportation problem is solved by using fuzzy NWCR, LCM and VAM methods. Finally, the result comparison table containing solution is given.

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