OPTIMUM SOLUTION TO FUZZY GAME THEORY PROBLEM USING TRIANGULAR FUZZY NUMBERS AND TRAPEZOIDAL FUZZY NUMBER

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Abstract:

The objective of this article is to introduce a new ranking method based on the area of membership function of fuzzy numbers. This new ranking method is used to find the best approximate solution to the fuzzy game theory problem. “Game theory provides a mathematical process for selecting an optimal strategy. In Artificial Intelligence, Fuzzy Mathematics and fuzzy logic are used to process natural language and are widely used in decision making. In real-life, game theory, analysis is used in economic competition, economic conditions such as negotiation, auctions, voting theory etc. However, in real life situations, the information available for decision making to select an optimum strategy is imprecise. In this article, the crisp game theory problem is transformed into a fuzzy game theory problem by using triangular and trapezoidal fuzzy numbers. To order any two fuzzy numbers, a new and simple method invented which is based on the area of membership function. A computer program was written in Python which is given in this article to make calculations easier and simpler.

Keywords:

Fuzzy, Triangular, Trapezoidal, Python, ranking, Saddle, Maximin, Minimax, Crisp, Strategy, Membership

Introduction:

Although the modern world sees significant changes in science and technology, part of the uncertainty cannot be avoided by any branch of science, engineering, medicine, and administration. It is well known that an important factor in the development of the modern concept of uncertainty was the publication of a seminar paper by Loft A. Zadeh in year 1965. In his article Zadeh transformed the probability theory and which is based on two value logic i.e. true or false. If ‘A’ is a fuzzy set and x is two valued logic, but it may be true to central degree to which x is realistically a member of A. The degree of membership lies between the interval [0,1]. The crisp set defined in such a way that we can classify it into two groups such as members and non-members. (Ali MahmodiNejada, January 2011) invented new method of Ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy number. (Mohamed A. H., Dec- 2020) introduced a New approach for ranking shadowed fuzzy numbers and Its application. (S. Salahshour S. Abbasbandy T., July 2011) used new techniques for ranking fuzzy numbers using fuzzy maximizing-minimizing points. (Savitha M T, 2017) make known to new methods for ranking of trapezoidal fuzzy nummbers. (Ganesan, 2018) used A new approach for the solution of fuzzy games using fuzzy numbers.

Game theory generally refers to the study of mathematical models that describe the behaviour of logical decision-makers. It is widely used in many fields such as economics, political science, politics, and computer science, and can be used to model many real-world scenarios. Game theory is a theoretical framework for conceiving social situations among competing players. Generally, a game refers to a situation involving a set of players who each have a set of possible choices, in which the outcome for any individual player depends partially on the choices made by other players.

Some Basic Definition:

1. **Fuzzy Set:**

   If X is a universe of discourse and x be any particular element of X, then a fuzzy set $\tilde{A}$ defined on X may be written as a collection of ordered pairs $\tilde{A}$ = $\{(x,\mu_{\tilde{A}}(x)) : x \in X\}$. Where each pair $(x,\mu_{\tilde{A}}(x))$ is called a singleton and $\mu_{\tilde{A}}(x)$ is membership function which maps X to [0,1]

2. **Support of a Fuzzy Set:**

   The support of a fuzzy set $\tilde{A}$ of the set X is a classical set defined as

   $$\mathrm{Sup}(\tilde{A}) = \{x \in X : \mu_{\tilde{A}}(x) > 0\}$$

3. **Fuzzy Number:**

   A Fuzzy set $\tilde{A}$ is a Fuzzy set on the real line R must be satisfy the following conditions

   a. There exist at least one $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0)=1$.
   b. $\mu_{\tilde{A}}(x)$ is piecewise continuous.
   c. $\tilde{A}$ must be normal and convex.

4. **Triangular Fuzzy Number:**

A triangular fuzzy number $\tilde{A}$ or simply triangular number represented with three points as follows $\tilde{A}$ = (a_1, a_2, a_3) hold the following conditions.

a. a_1 *to* a_2 membership function *is increasing function*

b. a_2 *to* a_3 membership function is decreasing function.

c. $a_1 \leq a_2 \leq a_3$

Its membership function is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x < a_2 \\ 1 & x = a_2 \\ \frac{(a_3-x)}{a_3-a_2} & a_2 < x \leq a_3 \end{cases}$$

5. **Trapezoidal Fuzzy Number:**

A Trapezoidal fuzzy number $\tilde{A}$ or simply trapezoidal number represented with four points as follows $\tilde{A}$ = $(a_1, a_2, a_{3,} a_4)$ hold the following conditions.

a. a_1 *to* a_2 membership function *is increasing function*

b. a_2 *to* a_3 membership function is 1.

c. a_3 *to* a_4 membership function is decreasing function.

d. $a_1 \leq a_2 \leq a_3 \leq a_4$

Its membership function is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & ; if\ a_1 \leq x < a_2 \\ 1 & ; if\ a_2 \leq x \leq a_3 \\ \frac{(a_4-x)}{a_4-a_3} & ; if\ a_3 < x \leq a_4 \end{cases}$$

6. **Crisp Set:**

A Crisp set is a special case of a fuzzy set, in which the membership function only takes two values, commonly defined as 0 and 1.

7. **Pure strategy:**

Pure strategy is a decision making rule in which one particular course of action is selected.

8. Mixed Strategy:

A set of strategies that a player chooses on a particular move of the game with some fixed probability are called mixed strategies.

9. **Saddle point:**

   If the maximin value equals to the minimax value, then the game is said to have a saddle point and the corresponding strategies which give the saddle point are called optimal strategies. The amount of payoff at an equilibrium point is called the crisp game value of the game matrix.

10. **Value of the game:**

   This is the expected payoff at the end of the game, when each player uses his optimal strategy.

11. **Solution of all 2× 2 matrix game**

   Consider the general 2x2 game matrix A=$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ To solve this game we proceed as follows:

   a. Test for a saddle point
   b. If there is no saddle point, solve by finding equalizing strategies. The optimal mixed strategies for player A=(p_1, p_2) *and for player* $B = (q_1, q_2)$

   Where $p_1 = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$, $p_2 = 1 - p_1$ and

$$q_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} \text{ , } q_2 = 1 - q_1$$

   Also Value of the game $V = \frac{a_{11}a_{22}-a_{21}a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

Ranking of Fuzzy Number

Let $\tilde{A}$ be a fuzzy number with $\mu_{\tilde{A}}$(x) is membership function which maps R to [0,1] and $Sup(\tilde{A}) = (a, b)$ is subset of R. The measure of $\tilde{A}$ is denoted by $R(\tilde{A})$ and defined as

$$R(\tilde{A}) = (a + b)\left[\frac{1}{b - a} * \ Area\ of\ membership\ function\ \mu_{\tilde{A}}(x) over\ [a, b]\right]$$

$$i.e.\ R(\tilde{A}) = (a + b)\left[\frac{1}{b - a}\int_a^b \mu_{\tilde{A}}(x)\ dx\right]$$

Ranking of Triangular Fuzzy Number:

Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number with $\mu_{\tilde{A}}$(x) is membership function and $Sup(\tilde{A}) = (a_1, a_3)$

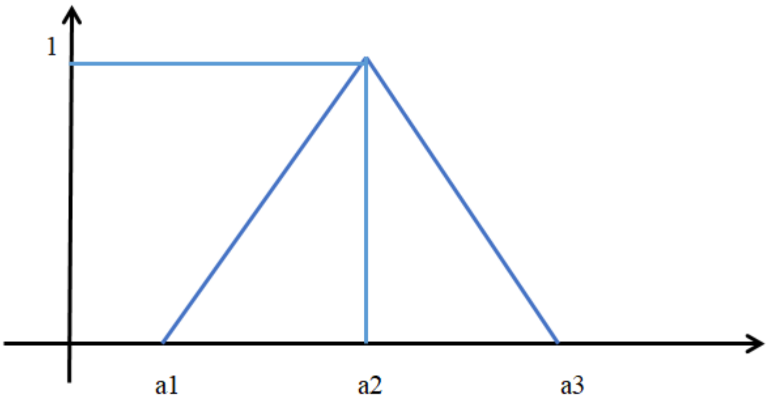

Fig 1: Triangular Fuzzy Number [a1, a2, a3]

$$Area\ of\ membership\ function\ \mu_{\tilde{A}}(x) over\ [a_1\ , a_3] = \frac{1}{2} \times 1 \times (a_3 - a_1)$$

$$\therefore Area\ of\ membership\ function\ \mu_{\tilde{A}}(x)\ over\ [a, b] = \frac{a_3 - a_1}{2}$$

$$\therefore R(\tilde{A}) = (a_1 + a_3)\left[\frac{1}{a_3 - a_1} \times \frac{a_3 - a_1}{2}\right]$$

$$\therefore R(\tilde{A}) = \frac{(a_1 + a_3)}{2}$$

Ranking of Trapezoidal Fuzzy Number

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number with $\mu_{\tilde{A}}$(x) is membership function and $Sup(\tilde{A}) = (a_1\ , a_4)$

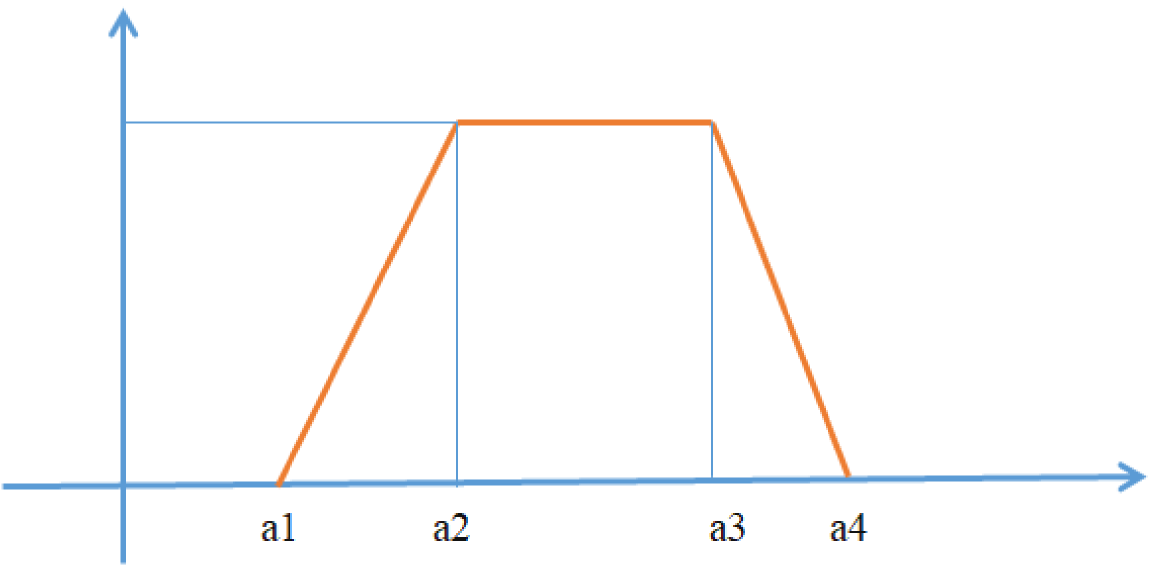

Fig 2: Trapezoidal Fuzzy Number

$$Area\ of\ membership\ function\ \mu_{\tilde{A}}(x) over\ [a_1\ , a_4]$$

$$= \left[\frac{1}{2} \times 1 \times (a_2 - a_1)\right] + [(a_3 - a_2) \times 1] + \left[\frac{1}{2} \times 1 \times (a_4 - a_3)\right]$$

$$\therefore Area\ of\ membership\ function\ \mu_{\tilde{A}}(x)\ over\ [a, b] = \frac{(a_3+a_4)-(a_1+a_2)}{2}$$

$$\therefore R(\tilde{A}) = (a_1 + a_4)\left[\frac{1}{a_4 - a_1} \times \frac{(a_3 + a_4) - (a_1 + a_2)}{2}\right]$$

$$\therefore R(\tilde{A}) = \frac{a_1 + a_4}{a_4 - a_1} \times \frac{(a_3 + a_4) - (a_1 + a_2)}{2}$$

$$\therefore R(\tilde{A}) = \frac{(a_1 + a_4)\ (a_3 + a_4 - a_1 - a_2)}{2(a_4 - a_1)}$$

Python Code for Ranking of Fuzzy Number:

```
n=int(input("Enter Your Choice for fuzzy number: 1:Triangular, 2: Trapezoidal="))
if(n==1):
   a1,a2,a3=[int(x) for x in input("Enter Your First Triagular Fuzzy number A=[a1,a2,a3]: ").split()]
   b1,b2,b3=[int(x) for x in input("Enter Your First Triagular Fuzzy number A=[b1,b2,b3]: ").split()]
   area1=(a1+a3)/2
   area2=(b1+b3)/2
   if(area1<=area2):
      print("A<<B")
   else:
      print("B<<A")
elif(n==2):
   a1,a2,a3,a4=[int(x) for x in input("Enter Your First Triagular Fuzzy number A=[a1,a2,a3,a4]: ").split()]
   b1,b2,b3,b4=[int(x) for x in input("Enter Your First Triagular Fuzzy number A=[b1,b2,b3,b4]: ").split()]
   area1=((a1+a4)*(a3+a4-a1-a2))/2
   area2=((b1+b4)*(b3+b4-b1-b2))/2
   if(area1<=area2):
      print("A<<B")
   else:
```

```
print("B<<A")
```

Some Output:

Enter Your Choice for fuzzy number: 1:Triangular, 2: Trapezoidal=1
Enter Your First Triagular Fuzzy number A=[a1,a2,a3]: 5 6 7
Enter Your First Triagular Fuzzy number A=[b1,b2,b3]: 1 2 3
B<<A

Enter Your Choice for fuzzy number: 1:Triangular, 2: Trapezoidal=2
Enter Your First Triagular Fuzzy number A=[a1,a2,a3,a4]: 1 2 3 4
Enter Your First Triagular Fuzzy number A=[b1,b2,b3,b4]: 3 4 5 6
A<<B

Examples:

1. Consider the following fuzzy game problem

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} (2,4,6) & (8,9,11) \\ (-2,0,3) & (-3,-1,1) \end{bmatrix} \end{array}$$

Solution:

By definition of Ranking of triangular fuzzy number: Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number with $\mu_{\tilde{A}}(x)$ is membership function and $Sup(\tilde{A}) = (a_1, a_3)$

$$R(\tilde{A}) = \frac{(a_1 + a_3)}{2}$$

Step 1:

Convert the given fuzzy problem into a crisp value problem

Fuzzy Number	**Crisp value**
$a_{11} = (2,4,6)$	$R(a_{11}) = 4$
$a_{12} = (8,9,11)$	$R(a_{12}) = 9.5$
$a_{21} = (-2,0,3)$	$R(a_{21}) = 0.5$
$a_{22} = (-3,-1,1)$	$R(a_{22}) = -1$

Step 2:

The pay-off matrix is

Player B

Player A $\begin{bmatrix} 4 & 9.5 \\ 0.5 & -1 \end{bmatrix}$

Minimum of 1^{st} row = 4, Minimum of 2^{st} row = -1, Maximum of 1^{st} column = 4

Maximum of 2^{st} column = 9.5 $\therefore Maximin = 4\ and\ Minimax = 4$

It has saddle point and ∴ Strategy for player A=A_1 and strategy $for\ player\ B = B_1$.

Also Value of the game $V = 4$

2. Consider the following fuzzy game problem

Player B

Player A $\begin{bmatrix} (2,4,5) & (-3,-2,1) \\ (-1,0,1) & (8,9,11) \end{bmatrix}$

Solution:

By definition of Ranking of triangular fuzzy number

Let $\tilde{A}$ = (a_1, a_2, a_3) be a triangular fuzzy number with $\mu_{\tilde{A}}$(x) is membership function and $Sup(\tilde{A}) = (a_1\ , a_3)$

$$R(\tilde{A}) = \frac{(a_1 + a_3)}{2}$$

Step 1:

Convert the given fuzzy problem into a crisp value problem

Fuzzy Number	Crisp value
$a_{11} = (2,4,5)$	$R(a_{11}) = 3.5$
$a_{12} = (-3,-2,1)$	$R(a_{12}) = -1$
$a_{21} = (-1,0,1)$	$R(a_{21}) = 0$
$a_{22} = (8,9,11)$	$R(a_{22}) = 9.5$

Step 2:

The pay-off matrix is

Player B

Player A $\begin{bmatrix} 3.5 & -1 \\ 0 & 9.5 \end{bmatrix}$

Minimum of 1^{st} row = -1, Minimum of 2^{st} row = 0, Maximum of 1^{st} column = 3.5

Maximum of 2^{st} column = 9.5 $\therefore Maximin = 0\ and\ Minimax = 3.5$

$$\therefore 0 \neq 3.5$$

It has no saddle point.

Step 3:

To find Optimum mixed strategy and value of the game

$$Here\ a_{11} = 3.5, a_{12} = -1, a_{21} = 0, a_{22} = 9.5$$

$p_1 = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{9.5-0}{(3.5+9.5)-(-1+0)} = \frac{9.5}{14} = \frac{19}{28};\ \ p_2 = 1 - \frac{19}{24} = \frac{9}{28}$

$q_1 = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{9.5-(-1)}{(3.5+9.5)-(-1+0)} = \frac{10.5}{14} = \frac{3}{4};\ \ q_2 = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore$ Strategy for player A= A=$(p_1, p_2) = \left(\frac{19}{28}, \frac{9}{28}\right)$

$\therefore$ Strategy $for\ player\ B = (q_1, q_2) = \left(\frac{3}{4}, \frac{1}{4}\right)$.

Also Value of the game $V = \frac{a_{11}a_{22}-a_{21}a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

$$V = \frac{(3.5 \times 9.5) - (0 \times (-1))}{(3.5 + 9.5) - (-1 + 0)} = \frac{33.25}{14} = \frac{19}{8}$$

$\therefore$ Value of the game $V = \frac{19}{8}$

3. Consider the following fuzzy game problem

Player B

$$\text{Player A} \quad \begin{bmatrix} (2,4,5,6) & (4,8,9,11) \\ (-2,0,3,4) & (-3,-1,0,1) \end{bmatrix}$$

Solution:

By definition of Ranking of trapezoidal fuzzy number

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number with $\mu_{\tilde{A}}(x)$ is membership function and $Sup(\tilde{A}) = (a_1, a_4)$

$$R(\tilde{A}) = \frac{(a_1 + a_4)\,(a_3 + a_4 - a_1 - a_2)}{2(a_4 - a_1)}$$

Step 1:

Convert the given fuzzy problem into a crisp value problem

Fuzzy Number	Crisp value
$a_{11} = (2,4,5,6)$	$R(a_{11}) = 5$
$a_{12} = (4,8,9,11)$	$R(a_{12}) = \frac{60}{7}$
$a_{21} = (-2,0,3,4)$	$R(a_{21}) = \frac{3}{2}$
$a_{22} = (-3,-1,0,1)$	$R(a_{22}) = -\frac{5}{4}$

Step 2:

The pay-off matrix is

Player B

Player A $\begin{bmatrix} 5 & \frac{60}{7} \\ \frac{3}{2} & -\frac{5}{4} \end{bmatrix}$

Minimum of 1^{st} row = 5, Minimum of 2^{st} row = $-\frac{5}{4}$, Maximum of 1^{st} column = 5

Maximum of 2^{st} column = $\frac{60}{7}$ $\therefore Maximin = 5\ and\ Minimax = 5$

∴It has saddle point

∴ Strategy for player A=A_1, ∴ Strategy $for\ player\ B = B_1$.

∴ Value of the game $V = 5$

4. Consider the following fuzzy game problem

Player B

Player A $\begin{bmatrix} (1,2,4,5) & (-3,-2,1,2) \\ (-1,0,1,2) & (7,8,9,11) \end{bmatrix}$

Solution:

By definition of Ranking of trapezoidal fuzzy number

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number with $\mu_{\tilde{A}}(x)$ is membership function and $Sup(\tilde{A}) = (a_1, a_4)$

$$R(\tilde{A}) = \frac{(a_1 + a_4)(a_3 + a_4 - a_1 - a_2)}{2(a_4 - a_1)}$$

Step 1:

Convert the given fuzzy problem into a crisp value problem

Fuzzy Number	Crisp value
$a_{11} = (1,2,4,5)$	$R(a_{11}) = \frac{9}{2}$
$a_{12} = (-3,-2,1,2)$	$R(a_{12}) = -\frac{4}{5}$
$a_{21} = (-1,0,1,2)$	$R(a_{21}) = \frac{2}{3}$
$a_{22} = (7,8,9,11)$	$R(a_{22}) = \frac{45}{4}$

Step 2:

The pay-off matrix is

Player B

$$\text{Player A} \quad \begin{bmatrix} \frac{9}{2} & -\frac{4}{5} \\ \frac{2}{3} & \frac{45}{4} \end{bmatrix}$$

Minimum of 1^{st} row = $-\frac{4}{5}$, Minimum of 2^{st} row = $\frac{2}{3}$, Maximum of 1^{st} column = $\frac{9}{2}$

Maximum of 2^{st} column = $\frac{45}{4}$

$\therefore Maximin = \frac{2}{3}$ and $Minimax = \frac{9}{2}$ $\therefore \frac{2}{3} \neq \frac{9}{2}$ Therefore it has no saddle point

Step 3:

To find Optimum mixed strategy and value of the game

$$Here\ a_{11} = \frac{9}{2}, a_{12} = -\frac{4}{5}, a_{21} = \frac{2}{3}, a_{22} = \frac{45}{4}$$

$$p_1 = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{\frac{45}{4}-\frac{2}{3}}{\left(\frac{9}{2}+\frac{45}{4}\right)-\left(-\frac{4}{5}+\frac{2}{3}\right)} = \frac{\frac{127}{12}}{\frac{953}{60}} = \frac{635}{953};\ p_2 = 1-\frac{635}{953} = \frac{318}{953}$$

$$q_1 = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{\frac{45}{4}-\left(-\frac{4}{5}\right)}{\left(\frac{9}{2}+\frac{45}{4}\right)-\left(-\frac{4}{5}+\frac{2}{3}\right)} = \frac{\frac{241}{20}}{\frac{953}{60}} = \frac{723}{953};\ q_2 = 1-\frac{723}{953} = \frac{230}{953}$$

∴ Strategy for player A= $(p_1, p_2) = \left(\frac{635}{953}, \frac{318}{953}\right)$

∴ Strategy $for\ player\ B = (q_1, q_2) = \left(\frac{723}{953}, \frac{230}{953}\right)$.

Also Value of the game $V = \frac{a_{11}a_{22}-a_{21}a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

$$V = \frac{\left(\frac{9}{2}\times\frac{45}{4}\right)-\left(\frac{2}{3}\times\left(-\frac{4}{5}\right)\right)}{\left(\frac{9}{2}+\frac{45}{4}\right)-\left(-\frac{4}{5}+\frac{2}{3}\right)} = \frac{\frac{6139}{120}}{\frac{953}{60}} = \frac{6139}{1906}$$

∴Value of the game $V = \frac{6139}{1906}$

Conclusion:

In this paper, to develop a method for solving problems using triangular fuzzy numbers and trapezoidal fuzzy numbers by using ranking of triangular and trapezoid fuzzy numbers. This article proposes a simple and concrete method that ranks triangular and trapezoidal fuzzy numbers. The Python code is used to get the exact ordering of triangular fuzzy numbers. This method gives a correct ranking order for the problems for decision-making problems under uncertainty calculation, since it is easy to calculate and gives acceptable results. Through a numerical example, we can conclude that the value obtained from fuzzy game theory from this method is optimum. In the future we want to extend our work to solve the fuzzy game problem using the Pentagonal fuzzy number, Octagonal fuzzy number.

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