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OPTIMUM SOLUTION TO FUZZY TRANSPORTATION PROBLEM USING DIFFERENT RANKING TECHNIQUESTO ORDER TRIANGULAR FUZZY NUMBERS

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ABSTRACT

Fuzzy numbers in fuzzy set theory cannot be compared but only partially ordered. But, when fuzzy numbers are used in real-world applications, then order of fuzzy numbers becomes significant. The objective of this article is to find best ranking methods to order triangular fuzzy number from the three (Center, Mean and α- Cut) methods. The three methods are used in this article to determine which of the following possibilities $\overline{M}<\overline{N}$, $\overline{M}>\overline{N}$ or $\overline{M}\approx\overline{N}$is true, for two fuzzy numbers $\overline{M}$, $\overline{N}$ to obtain approximate solutions to fuzzy optimization problems. Error which is difference between crisp transportation problem and fuzzy transportation problem is obtained using these three ranking methods. Comparison table is used to find best ranking method to fuzzy transportation problem.

Keywords: Fuzzy Transportation, Ranking, α- Cut, LCM, Ordered

INTRODUCTION

Crisp transportation problems converted into fuzzy transportation problems with triangular fuzzy numbers. There are many methods of fuzzy numbers say qualitative, quantitative and based on α–cuts to the ordering relation between any two fuzzy numbers. to compare any two fuzzy numbers there is not a universally accepted method in fuzzy set theory. The method considers the overall possibility distributions of fuzzy numbers in their evaluations for ranking and provides users with a method of changing viewpoints for evaluationsis introduced by Lee-Kwang(1999), Ranking Fuzzy Numbers with a Distance Method using Circumcenter of Centroids and an Index of Modality given byP.PhaniBushanRao (2011),Ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy number is developed by MahmodiNejad(2011), Solution of Fuzzy Game Problem Using Triangular Fuzzy Number method given by R. Senthil Kumar(2015), New Methods for Ranking of Trapezoidal Fuzzy Numbers is introduced by Savitha M T(2017), Sub Interval Average Method for Ranking of Linear Fuzzy Numbers introduced by Stephen Dinagar(2017).

In this article using three ranking methods are applied to obtained initial basic feasible solution to fuzzy transportation problem.

BASIC DEFINITIONS:

Triangular Fuzzy Number:

Fuzzy number represented with three points (a_1, a_2, a_3) holds the following conditions called Triangular fuzzy number

i) a_1 to a_2 is increasing function

ii) a_2 to a_3 is decreasing function

iii) $a_1 \leq a_2 \leq a_3$

Its membership function is defined as follows

$$\mu_A(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a1)} & a_1 \leq x < a_2 \\ 1 & x = a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & a_2 < x \leq a_3 \end{cases}$$

$$\mu_A(x)$$

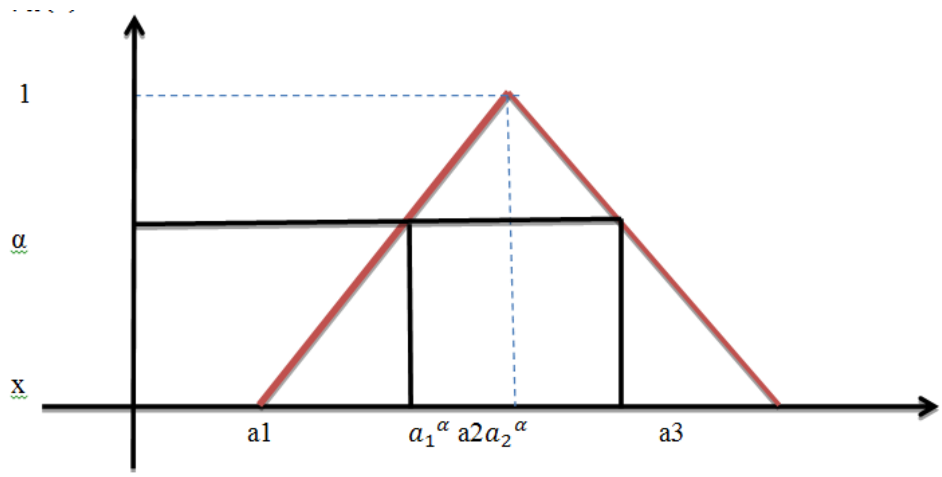

Figure1: Triangular fuzzy number $[a_1, a_2, a_3]$

α- Cut for triangular fuzzy number:
For any α ∈ [0, 1] from

$\frac{a_1^\alpha - a_1}{a_2 - a_1} = \alpha, \frac{a_3 - a_3^\alpha}{a_3 - a_2} = \alpha$

$a_1^\alpha = (a_2 - a_1)\alpha + a_1$, $a_3^\alpha = = -(a_3 - a_2)\alpha + a_3$

Thus $\bar{A}_\alpha = [a_1^\alpha, a_3^\alpha] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3]$

Operations on triangular fuzzy number:
Addition, Subtraction and Multiplication of any two triangular fuzzy numbers are also triangular fuzzy number. Suppose triangular fuzzy numbers $\bar{A}$ and $\bar{B}$ are defined as,

$\bar{A} = (a_1, a_2, a_3)$ and $\bar{B} = (b_1, b_2, b_3)$

i) Addition $\bar{A}$ (+)$\bar{B}$ = $(a_1, a_2, a_3) + (b_1, b_2, b_3)$

$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

ii) Subtraction $\bar{A}$ (-)$\bar{B}$ = $(a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

iii) Symmetric image: $(-\bar{A}) = (-a_3, -a_2, -a_1)$

iv) Multiplication:

$\bar{A} \times \bar{B} = (\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$.

Ordering (Ranking) of two Triangular fuzzy numbers: To order any two fuzzy triangular number following three methods are used

Method 1: α- Cut Method
To order any two triangular fuzzy numbers $\bar{X} = [a_1, a_2, a_3]$and $\bar{Y} = [b_1, b_2, b_3]$we find here α cut say $\bar{X}_\alpha$ and $\bar{Y}_\alpha$. Thus $\bar{X}_\alpha = [a_1^\alpha, a_2^\alpha]$ and $\bar{X}_\alpha = [b_1^\alpha, b_2^\alpha]$

Then $\bar{X} \leq \bar{Y}$ if $a_1^\alpha \leq b_1^\alpha$ and $a_2^\alpha \leq b_2^\alpha$otherwise $\bar{Y} \leq \bar{X}$

Method 2: Center Method
To order any two triangular fuzzy numbers $\bar{X} = [a_1, a_2, a_3]$and $\bar{Y} = [b_1, b_2, b_3]$we find the center of fuzzy number $\bar{X}$ which is a_2 similarly center of fuzzy number $\bar{Y}$which is b2 Then $\bar{X} \leq \bar{Y}$if $a_2 \leq b_2$otherwise $\bar{Y} \leq \bar{X}$

Method 3: Average Method : To order any two triangular fuzzy numbers $\bar{X} = [a_1, a_2, a_3]$and $\bar{Y} = [b_1, b_2, b_3]$we find average of fuzzy number $\bar{X}$ say $Xave = \frac{(a1+a2+a3)}{3}$ and average of fuzzy number $\bar{Y}$ say $Yave = \frac{(b1+b2+b3)}{3}$
Then $\bar{X} \leq \bar{Y}$ if $Xave \leq Yave$otherwise $\bar{Y} \leq \bar{X}$

Mat-lab Code:
/To find addition, subtraction, multiplication of fuzzy triangular numbers

close all

```
clear all
clc
'Enter Your first Triangular fuzzy number\n'
a1=input('');
b1=input('');
c1=input('');
a=[a1  b1  c1]
'Enter Your second Triangular fuzzy number\n'
a2=input('');
b2=input('');
c2=input('');
b=[a2  b2  c2]
'Enter Your Choice 1= Addition 2= Subtraction  3 = Multiplication ,4 = order'
ch=input('');
switch(ch)
case 1
'Addition of two fuzzy Pentagonal number is'
c = a + b
case 2
'subtraction of two fuzzy Pentagonal is(a-b)'
c=[a1-c2  b1-b2  c1-a2]
case 3
'Multiplication of two fuzzy Pentagonalis(a-b)'
c=[a1*a2  b1*b2  c1*c2]
otherwise
disp('unknown method')
end
```

DEFUZZIFICATION:

If $\bar{x}$ = (*a*, *b*, *c*) be any given triangular fuzzy number then we use mean method to Defuzzifyi.e. $x = \frac{a+b+c}{3}$.

If (-*a*, 0, *a*) is given fuzzy triangular number then its crisp value is zero.

Method for solving fuzzy transportation problem:

In this section, the crisp transportation problem is converted into fuzzy transportation problem by using Triangular, Fuzzy numbers. Fuzzy Least Cost Method (FLCM) is applied to obtain initial basic feasible solution.

Fuzzy LeastCost Method (FLCM):

Suppose there are *m* factories and *n* warehouses then transportation problem is usually represented in tabular form

Table 1: Fuzzy transportation problem

Destination \ Origin	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$	----------------------	$\bar{D}_n$	Supply

$\bar{\boldsymbol{O}}_1$	$\bar{C}_{11}$	$\bar{C}_{12}$	$\bar{C}_{13}$	----------	$\bar{C}_{1n}$	$\bar{A}_1$
$\bar{\boldsymbol{O}}_2$	$\bar{C}_{21}$	$\bar{C}_{22}$	$\bar{C}_{23}$	----------	$\bar{C}_{2n}$	$\bar{A}_2$
⋮	⋮	⋮	⋮		⋮	⋮
$\bar{\boldsymbol{O}}_m$	$\bar{C}_{m1}$	$\bar{C}_{m2}$	$\bar{C}_{m3}$	----------	$\bar{C}_{mn}$	$\bar{A}_m$
Demand	$\bar{B}_1$	$\bar{B}_2$	$\bar{B}_3$		$\bar{B}_n$	$\sum_{i=1}^{n} \bar{B}_i = \sum_{j=1}^{m} \bar{A}_j$

Step 1: Decide the smallest fuzzy cost in fuzzy transportation table. Let it be $\bar{c}_{ij.}$ Find $\bar{x}_{ij}$ = minimum $(\bar{A}_\iota, \bar{B}_J)$. The following three conditions may arise:

Condition (*i*): If minimum $(\bar{A}_\iota, \bar{B}_J) = \bar{A}_\iota$ then allocate $\bar{x}_{ij} = \bar{A}_\iota$ in the NWC of $m \times n$ fuzzy transportation table.Ignore the i^{th}row to obtain a new fuzzy transportation table of order (m-1) × n. Replace $\bar{B}_J$ by $\bar{B}_J - \bar{A}_\iota$ in the obtained fuzzy transportation table. Go to Step 2.

Condition (*ii*): If minimum $(\bar{A}_\iota, \bar{B}_J) = \bar{B}_J$ then allocate $\bar{x}_{ij} = \bar{B}_J$ in the NWC of $m \times n$ fuzzy transportation table.Ignore the j^{th} row to obtain a new fuzzy transportation table of order m × (n-1). Replace $\bar{A}_\iota$ by $\bar{A}_\iota - \bar{B}_J$ in the obtained fuzzy transportation table. Go to

Step2, Condition (*iii*): If $\bar{A}_\iota = \bar{B}_J$ then either follow condition(*i*) or condition (*ii*) but not both simultaneously. Go to Step 2.

Step2: Repeat Step 1 for the obtained fuzzy transportation table, until the fuzzy transportation table is reduced into a fuzzy transportation table of order 1 × 1**.**

Step3: Assign all $\bar{x}_{ij}$ in the ij^{th} cell of the given fuzzy transportation table.

Step 4: The obtained IFBFS and initial fuzzy transportation cost are $\bar{x}_{ij}$and $\sum_{i=1}^{m}\sum_{j=1}^{n} \bar{x}_{ij}\, \bar{c}_{ij}$respectively.

Numerical example:
Consider the following crisp transportation problem

Numerical example:
Consider the following crisp transportation problem

Table 2: Crisp transportation problem

Destination Origin	$\bar{C}_1$	$\bar{C}_2$	$\bar{C}_3$	$\bar{C}_4$	Supply
$\bar{O}_1$	9	6 **(12)**	3 **(15)**	7	27
$\bar{O}_2$	5 **(9)**	8 **(1)**	3	6	10
$\bar{O}_3$	8	11 **(1)**	6	2 **(13)**	14
Demand	9	14	15	13	

Minimum Transportation Cost = 12*6 + 15*3 + 9*5 + 1*8 + 1*11 + 13*2 = 207

Table 3: Fuzzy transportation problem with ranking centre method

Destination Origin	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$	$\bar{D}_4$	Supply
$\bar{O}_1$	[7, 9, 11]	[4,6,8] **(8,12,16)**	[1,3,5] **(13,15,17)**	[5,7,9]	[25,27,29]
$\bar{O}_2$	[3,5,7] **(7,9,11)**	[6,8,10] **(-3,1,5)**	[1,3,5]	[4,6,8]	[8,10,12]
$\bar{O}_3$	[6,8,10]	[9,11,13] **(-3,1,5)**	[4,6,8]	[0,2,4] **(11,13,15)**	[12,14,16]
Demand	[7,9,11]	[12,14,16]	[13,15,17]	[11,13,15]	

Minimum Transportation Cost = [21, 45, 77] + [32, 72,128] + [13, 45, 85] + [-8, 8, 50] + [-27, 11, 65] + [0, 26,60] = [31, 207, 465]

Final computational value of fuzzy triangular number is**[-6, 0, 6]** = 0

Table 3: Fuzzy transportation problem with ranking Average method

Destination / Origin	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$	$\bar{D}_4$	Supply
$\bar{O}_1$	[7, 9, 11]	[4,6,8] **(9,13,17)**	[1,3,5] **(12,14,16)**	[5,7,9]	[25,27,29]
$\bar{O}_2$	[3,5,7] **(7,9,11)**	[6,8,10]	[1,3,5] **(-4,1,3)**	[4,6,8]	[8,10,12]
$\bar{O}_3$	[6,8,10]	[9,11,13] **(-5,1,7)**	[4,6,8]	[0,2,4] **(11,13,15)**	[12,14,16]
Demand	[7,9,11]	[12,14,16]	[13,15,17]	[11,13,15]	

Minimum Transportation Cost = [21, 45, 77] + [36, 78,136] + [-45, 11, 91] + [13, 42, 85] + [-4, 3, 15] + [0, 26,60] = [21, 205, 464]

Final computational value of fuzzy triangular number is**[-9, 0, 9]** = 0

Table 4: Fuzzy transportation problem with ranking α- Cut method

Destination / Origin	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$	$\bar{D}_4$	Supply
$\bar{O}_1$	[7, 9, 11]	[4,6,8] **(12,14,16)**	[1,3,5] **(11,13,15)**	[5,7,9]	[25,27,29]
$\bar{O}_2$	[3,5,7] **(2,8,14)**	[6,8,10]	[1,3,5] **(-2,2,6)**	[4,6,8]	[8,10,12]
$\bar{O}_3$	[6,8,10] **(-3,1,5)**	[9,11,13]	[4,6,8]	[0,2,4] **(11,13,15)**	[12,14,16]
Demand	[7,9,11]	[12,14,16]	[13,15,17]	[11,13,15]	

Minimum Transportation Cost = [6, 40, 98] + [48, 84,128] + [11, 39, 75] + [-2, 6, 30] + [-18, 8, 50] + [0, 26,60] = [45,203,441]

Final computational value of fuzzy triangular number is**[-12, 0, 12]** = 0

ERROR:

The error value is obtained using following formula:

Error = |Crisp transportation problem – Fuzzy Transportation Problem|

Table 5: Error

Ranking Method	**FLCM**	**Defuzzification**	**Error**
		Mean Method	**Mean Method**
Center	[31, 207, 465]	234.333	27
Area	[21, 205, 464]	230.000	23
α-cut	[45,203,441]	229.666	22.666

CONCLUSION AND FUTURE WORK

In This article convert the crisp transportation problem is converted into a fuzzy transportation problem. Initial basic feasible solution obtained by using Least Cost Method. Three ranking method are used to order fuzzy triangular numbers. The mat-lab code is used to get exact ordering of triangular fuzzy number. Apply this ranking method to obtained solution to fuzzy transportation problem.From numerical example, we can conclude that α- Cut gives minimum fuzzy transportation cost. In Future we want to extend our work by applying this ranking technique to fuzzy optimization techniques such as Network Analysis, Assignment Problem, queuing theory etc.

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